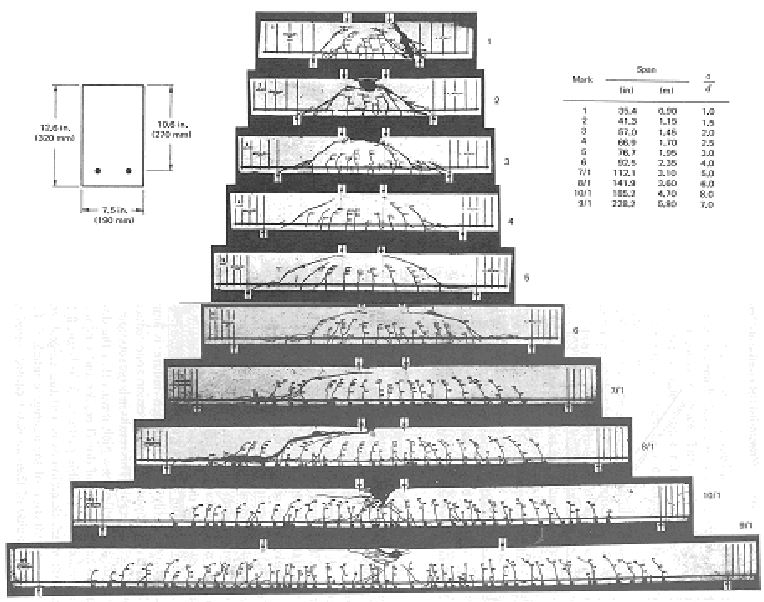
Lecture Notes in:

Mechanics and Design of REINFORCED CONCRETE



Victor E. Saouma

Dept. of Civil Environmental and Architectural Engineering

University of Colorado, Boulder, CO 80309-0428

Contents

List of Figures

List of Tables

Chapter 1

INTRODUCTION

1.1 Material

1.1.1 Concrete

`This section is adapted from` *Concrete* `by Mindess and Young, Prentice Hall, 1981`

1.1.1.1 Mix Design

1.1.1.1.1 Constituents

1 Concrete is a mixture of Portland cement, water, and aggregates (usually sand and crushed stone).

2 Portland cement is a mixture of calcareous and argillaceous materials which are calcined in a kiln and then pulverized. When mixed with water, cement hardens through a process called **hydration**.

3 Ideal mixture is one in which:

1. A minimum amount of cement-water paste is used to fill the interstices between the particles of aggregates.
2. A minimum amount of water is provided to complete the chemical reaction with cement. Strictly speaking, a water/cement ratio of about 0.25 is needed to complete this reaction, but then the concrete will have a very low "workability".

In such a mixture, about 3/4 of the volume is constituted by the aggregates, and the remaining 1/4 being the cement paste.

4 Smaller particles up to 1/4 in. in size are called **fine aggregates**, and the larger ones being **coarse aggregates**.

5 Portland Cement has the following ASTM designation

I Normal

II Moderate sulfate resistant, moderate heat of hydration

III High early strength (but releases too much heat)

ASTM Design.	Size	
	mm	in.
Coarse Aggregate		
3 in.	75	3
$2^{1/2}$ in.	63	2.5
2 in.	50	2
$1^{1/2}$ in.	37.5	1.5
1 in.	25	1
3/4 in.	19	0.75
1/2 in.	12.5	0.50
3/8 in.	9.5	0.375
Fine Aggregate		
No. 4	4.75	0.187
No. 8	2.36	0.0937
No. 16	1.18	0.0469
No. 30	0.60 (600 μm)	0.0234
No. 50	300 μm	0.0124
No. 100	150 μm	0.0059

Table 1.1: ASTM Sieve Designation's Nominal Sizes Used for Concrete Aggregates

Sieve Size	% Passing Each Sieve (Nominal Maximum Size)			
	$1^{1/2}$ in.	1 in.	3/4 in.	1/2 in.
$1^{1/2}$ in.	95-100	100	-	-
1 in.	-	95-100	100	-
3/4 in.	35-70	-	90-100	100
1/2 in.	-	25-60	-	90-100
3/8 in.	10-30	-	20-55	40-70
No. 4	0-5	0-10	0-10	0-15
No. 8	-	0-5	0-5	0-5

Table 1.2: ASTM C33 Grading Limits for Coarse Concrete Aggregates

Sieve Size	% Passing
3/4 in.	100
No. 4	95-100
No. 8	80-100
No. 16	50-85
No. 30	25-60
No. 50	10-30
No. 100	2-10

Table 1.3: ASTM C33 Grading Limits for Fine Concrete Aggregates

Sieve Size	Weight Retained (g)	Amount Retained (wt. %)	Cumulative Amount Retained (%)	Cumulative Amount Passing (%)
No. 4	9	2	2	98
No. 8	46	9	11	89
No. 16	97	19	30	70
No. 30	99	20	50	50
No. 50	120	24	74	26
No. 100	91	18	92	8
Sample Weight 500 g.			$\sum = 259$	
Fineness modulus=259/100=2.59				

Table 1.4: Example of Fineness Modulus Determination for Fine Aggregate

1.1.1.1.2 Preliminary Considerations

24 There are two fundamental aspects to mix design to keep in mind:

1. **Water/Cement ratio:** where the strength is inversely proportional to the water to cement ratio, approximately expressed as:

$$f'_c = \frac{A}{B^{1.5w/c}} \tag{1.4}$$

For f'_c in psi, A is usually taken as 14,000 and B depends on the type of cement, but may be taken to be about 4. It should be noted that w/c controls not only the strength, but also the **porosity** and hence the **durability**.

2. **Aggregate Grading:** In order to minimize the amount of cement paste, we must maximize the volume of aggregates. This can be achieved through proper packing of the granular material. The "ideal" grading curve (with minimum voids) is closely approximated by the Fuller curve

$$P_t = \left(\frac{d}{D}\right)^q \tag{1.5}$$

where P_t is the fraction of total solids finer than size d, and D is the maximum particle size, q is generally taken as 1/2, hence the *parabolic* grading.

1.1.1.1.3 Mix procedure

25 Before starting the mix design process, the following **material properties** should be determined:

1. Sieve analysis of both fine and coarse aggregates
2. Unit weight of the coarse aggregate
3. Bulk specific gravities
4. absorption capacities of the aggregates

Slump in.	Sizes of Aggregates				
	3/8 in.	1/2 in.	3/4 in.	1 in.	$1^{1/2}$ in.
Non-Air-Entrained Concrete					
1-2	350	335	315	300	275
3-4	385	365	340	325	300
6-7	410	385	360	340	315
Air-Entrained Concrete					
1-2	305	295	280	270	250
3-4	340	325	305	295	275
6-7	365	345	325	310	290

Table 1.7: Approximate Mixing Water Requirements, lb/yd^3 of Concrete For Different Slumps and Nominal Maximum Sizes of Aggregates

28 days f'_c	w/c Ratio by Weight	
	Non-air-entrained	Air-entrained
6,000	0.41	-
5,000	0.48	0.40
4,000	0.57	0.48
3,000	0.68	0.59
2,000	0.82	0.74

Table 1.8: Relationship Between Water/Cement Ratio and Compressive Strength

Fine Aggregates: Bulk specific gravity (SSD) = 2.65; absorption capacity = 1.3 %; Total moisture content=5.5%; fineness modulus = 2.70

The sieve analyses of both the coarse and fine aggregates fall within the specified limits. With this information, the mix design can proceed:

1. Choice of **slump** is consistent with Table 1.5.

2. **Maximum aggregate size** (3/4 in) is governed by reinforcing details.

3. Estimation of mixing **water**: Because water will be exposed to freeze and thaw, it must be air-entrained. From Table 1.6 the air content recommended for extreme exposure is 6.0%, and from Table 1.7 the water requirement is 280 lb/yd^3

4. From Table 1.8, the **water to cement ratio** estimate is 0.4

5. **Cement content**, based on steps 4 and 5 is 280/0.4=700 lb/yd^3

6. **Coarse aggregate content**, interpolating from Table 1.9 for the fineness modulus of the fine aggregate of 2.70, the volume of dry-rodded coarse aggregate per unit volume of concrete is 0.63. Therefore, the coarse aggregate will occupy $0.63 \times 27 = 17.01$ ft^3/yd^3. The OD weight of the coarse aggregate is 17.01 ft^3/yd^3, $\times$ 100 lbs/ft^3=1,701 lb. The SSD weight is 1,701 $\times$ 1.01=1,718 lb.

7. **Fine aggregate content** Knowing the weights and specific gravities of the water, cement, and coarse aggregate, and knowing the air volume, we can calculate the volume per yd^3 occupied by the different ingredients.

Water	280/62.4	=	4.49	ft^3
Cement	700/(3.15)(62.4)	=	3.56	ft^3
Coarse Aggregate (SSD)	1,718/(2.70)(62.4)	=	1.62	ft^3
Air	(0.06)(27)	=	1.62	ft^3
			19.87	ft^3

Hence, the fine aggregate must occupy a volume of $27.0 - 19.87 = 7.13$ ft^3. The required SSD weight of the fine aggregate is 7.13 ft^3 (2.65)(62.4)lb/ft^3 =1,179 lbs lb.

8. **Adjustment for moisture** in the aggregate. Since the aggregate will be neither SSD or OD in the field, it is necessary to adjust the aggregate weights for the amount of water contained in the aggregate. Only surface water need be considered; absorbed water does not become part of the mix water. For the given moisture contents, the adjusted aggregate weights become:

Coarse aggregate (wet)=1,718(1.025-0.01) = 1,744 lb/yd^3 of dry coarse
Fine aggregate (wet)=1,179(1.055-0.013) = 1,229 lb/yd^3 of dry fine

Surface moisture contributed by the coarse aggregate is 2.5-1.0 = 1.5%; by the fine aggregate: 5.5-1.3 = 4.2%; Hence we need to decrease water to
280-1,718(0.015)-1,179(0.042) = 205 lb/yd^3.

Thus, the estimated batch weight per yd^3 are

Chapter 2

FLEXURE

1 This is probably the longest chapter in the notes, we shall cover in great details flexural design/analysis of R/C beams starting with uncracked section to failure conditions.

1. Uncracked elastic (uneconomical)
2. cracked elastic (service stage)
3. Ultimate (failure)

2.1 Uncracked Section

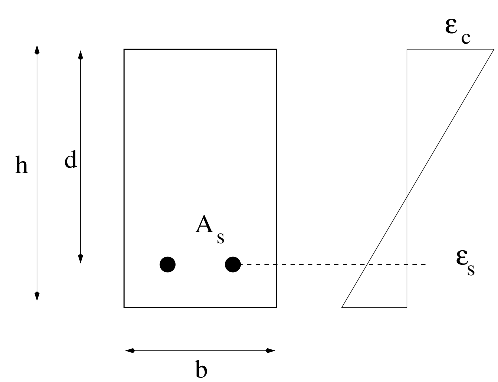


Figure 2.1: Strain Diagram Uncracked Section

2 Assuming perfect bond between steel and concrete, we have $\varepsilon_s = \varepsilon_c$, Fig. 2.1

$$\varepsilon_s = \varepsilon_c \Rightarrow \frac{f_s}{E_s} = \frac{f_c}{E_c} \Rightarrow f_s = \frac{E_s}{E_c} f_c \Rightarrow f_s = n f_c \tag{2.1}$$

where n is the modular ratio $n = \frac{E_s}{E_c}$

3 Tensile force in steel $T_s = A_s f_s = A_s n f_c$

4 Replace steel by an equivalent area of concrete, Fig. 2.2.

$$f_{ct} = \frac{Mc}{I} = \frac{(540,000) \text{ lb.in}(25-13.2) \text{ in}}{(14,722) \text{ in}^4} = \boxed{433 \text{ psi}} < 475 \text{ psi}\surd \quad (2.3\text{-h})$$

$$f_s = n\frac{Mc}{I} = (8)\frac{(540,000)(23-13.2) \text{ in}}{(14,722)} = \boxed{2,876 \text{ psi}} \quad (2.3\text{-i})$$

■

2.2 Section Cracked, Stresses Elastic

7 This is important not only as an acceptable alternative ACI design method, but also for the later evaluation of crack width under service loads.

2.2.1 Basic Relations

8 If $f_{ct} > f_r$, $f_{cc} <\approx .5f'_c$ and $f_s < f_y$ we will assume that the crack goes all the way to the N.A and we will use the transformed section, Fig. 2.3

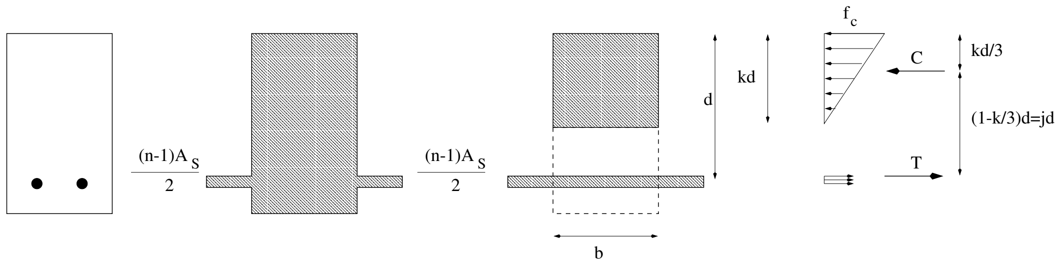


Figure 2.3: Stress Diagram Cracked Elastic Section

9 To locate N.A, tension force = compressive force (by def. NA) (Note, for linear stress distribution and with $\Sigma F_x = 0; \sigma = by \Rightarrow \int bydA = 0$, thus $b\int ydA = 0$ and $\int ydA = \overline{y}A = 0$, by definition, gives the location of the neutral axis)

10 Note, N.A. location depends only on geometry & $n\left(\frac{E_s}{E_c}\right)$

11 Tensile and compressive forces are equal to $C = \frac{bkd}{2}f_c$ & $T = A_sf_s$ and neutral axis is determined by equating the moment of the tension area to the moment of the compression area

$$b(kd)\left(\frac{kd}{2}\right) = nA_s(d-kd) \qquad 2^{nd} \text{ degree equation} \quad (2.4\text{-a})$$

$$M = Tjd = A_sf_sjd \Rightarrow f_s = \frac{M}{A_sjd} \quad (2.4\text{-b})$$

$$M = Cjd = \frac{bkd}{2}f_cjd = \frac{bd^2}{2}kjf_c \Rightarrow \boxed{f_c = \frac{M}{\frac{1}{2}bd^2kj}} \quad (2.4\text{-c})$$

where $j = (1-k/3)$.

Review Start by determining ρ,

- If $\rho < \rho_b$ steel reaches max. allowable value before concrete, and

$$\boxed{M = A_s f_s jd} \tag{2.9}$$

- If $\rho > \rho_b$ concrete reaches max. allowable value before steel and

$$M = f_c \frac{bkd}{2} jd \tag{2.10}$$

or

$$\boxed{M = \frac{1}{2} f_c jkbd^2 = Rbd^2} \tag{2.11}$$

where

$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$

Design We define

$$R \stackrel{\text{def}}{=} \frac{1}{2} f_c kj \tag{2.12}$$

where $k = \frac{n}{n+r}$, solve for bd^2 from

$$bd^2 = \frac{M}{R} \tag{2.13}$$

assume b and solve for d. Finally we can determine A_s from

$$A_s = \rho_b bd \tag{2.14}$$

17 Summary

Review	**Design**
$b, d, A_s \surd$	$M \surd$
$M?$	$b, d, A_s?$
$\rho = \frac{A_s}{bd}$	$k = \frac{n}{n+r}$
	$j = 1 - \frac{k}{3}$
$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$	$r = \frac{f_s}{f_c}$
$r = \frac{f_s}{f_c}$	$R = \frac{1}{2} f_c kj$
$\rho_b = \frac{n}{2r(n+r)}$	$\rho_b = \frac{n}{2r(n+r)}$
$\rho < \rho_b \quad M = A_s f_s jd$	$bd^2 = \frac{M}{R}$
$\rho > \rho_b \quad M = \frac{1}{2} f_c bkd^2 j$	$A_s = \rho_b bd$ or $A_s = \frac{M}{f_s jd}$

■ **Example 2-2: Cracked Elastic Section**

Solution:

$$\rho = \frac{A_s}{bd} = \frac{2.35}{(10)(23)} = .0102 \quad (2.16\text{-a})$$

$$f_s = 24 \text{ ksi} \quad (2.16\text{-b})$$

$$f_c = (.45)(4,000) = 1,800 \text{ psi} \quad (2.16\text{-c})$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(.0102)8 + (.0102)^2} - (8)(.0102) = .331 \quad (2.16\text{-d})$$

$$j = 1 - \frac{k}{3} = .889 \quad (2.16\text{-e})$$

$$N.A. \; @ \; (.331)(23) = 7.61 \text{ in} \quad (2.16\text{-f})$$

$$\rho_b = \frac{n}{2r(n+r)} = \frac{8}{(2)(13.33)(8+13.33)} = .014 > \rho \Rightarrow \quad \text{Steel reaches elastic limit} \quad (2.16\text{-g})$$

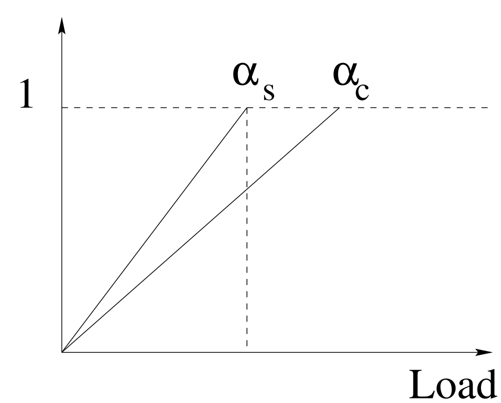
$$M = A_s f_s j d = (2.35)(24)(.889)(23) = \boxed{1,154 \text{ k.in} = 96 \text{ k.ft}} \quad (2.16\text{-h})$$

Note, had we used the alternate equation for moment (wrong) we would have overestimated the design moment:

$$M = = \frac{1}{2} f_c b k d^2 j \quad (2.17\text{-a})$$

$$= \frac{1}{2}(1.8)(10)(0.33)(0.89)(23)^2 = 1,397 \text{ k.in} > 1,154 \text{ k.in} \quad (2.17\text{-b})$$

If we define $\alpha_c = f_c/1,800$ and $\alpha_s = f_s/24,000$, then as the load increases both α_c and α_s increase, but at different rates, one of them α_s reaches 1 before the other.



■

■ Example 2-4: Working Stress Design Method; Design

Design a beam to carry $LL = 1.9$ k/ft, $DL = 1.0$ k/ft with $f'_c = 4,000$ psi, $f_y = 60,000$ psi, $L = 32$ ft.

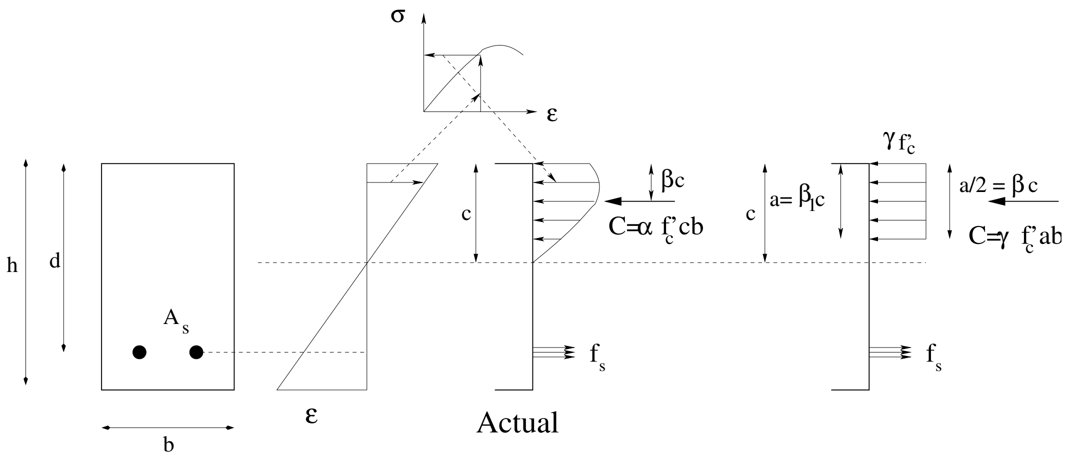


Figure 2.5: Cracked Section, Limit State

$$\alpha = \frac{f_{av}}{f'_c} \tag{2.20-b}$$

$$a = \beta_1 c \tag{2.20-c}$$

Thus

$$\gamma = \frac{\alpha}{\beta_1} \tag{2.21}$$

But the location of the resultant forces must be the same, hence

$$\beta_1 = 2\beta \tag{2.22}$$

21 From Experiments

f'_c (psi)	<4,000	5,000	6,000	7,000	8,000
α	.72	.68	.64	.60	.56
β	.425	.400	.375	.350	.325
$\beta_1 = 2\beta$	.85	.80	.75	.70	.65
$\gamma = \alpha/\beta_1$	0.85	0.85	0.85	0.86	0.86

22 Thus we have, (**ACI-318 10.2.7.3**):

$$\boxed{\begin{array}{lll} \beta_1 = & .85 & \text{if } f'_c \leq 4,000 \\ = & .85 - (.05)(f'_c - 4,000)\frac{1}{1,000} & \text{if } 4,000 < f'_c < 8,000 \end{array}} \tag{2.23}$$

23 Failure can occur by either

yielding of steel: $\varepsilon_s = \varepsilon_y$; Progressive

crushing of concrete: $\varepsilon_c = .003$; Sudden; **(ACI 10.3.2)**.

27 Also we need to specify a minimum reinforcement ratio

$$\boxed{\rho_{min} \geq \frac{200}{f_y} \qquad \textbf{(ACI 10.5.1)}} \tag{2.29}$$

to account for temperature & shrinkage

28 Note, that ρ need not be as high as $0.75\rho_b$. If steel is relatively expensive, or deflection is of concern, can use lower ρ.

29 As a rule of thumb, if $\rho < 0.5\rho_b$, there is no need to check for deflection.

2.3.3 Review

30 Given, b, d, A_s, f'_c, f_y, determine the moment capacity M.

$$\begin{aligned} \rho_{act} &= \frac{A_s}{bd} \\ \rho_b &= (.85)\beta_1 \frac{f'_c}{f_y}\frac{87}{87+f_y} \end{aligned} \tag{2.30}$$

- $\rho_{act} < \rho_b$: Failure by yielding and

$$\boxed{\begin{array}{rcll} a &=& \frac{A_s f_y}{.85 f'_c b} & \Sigma F_x = 0 \\ M_d &=& \phi A_s f_y (d - \frac{a}{2}) & \Sigma M = 0 \end{array}} \tag{2.31}$$

- $\rho_{act} > \rho_b$ is not allowed by code, in this case we have an extra unknown f_s.

31 We now have one more unknown f_s, and we will need an additional equation (from strain diagram).

$$\boxed{\begin{array}{rcll} c &=& \frac{A_s f_s}{.85 f'_c b \beta_1} & \Sigma F_x = 0 \\ \frac{c}{d} &=& \frac{.003}{.003+\varepsilon_s} & \text{From strain diagram} \\ M_d &=& \phi A_s f_s (d - \frac{\beta_1 c}{2}) & \Sigma M = 0 \end{array}} \tag{2.32}$$

We can solve by iteration, or substitution and solution of a quadratic equation.

2.3.4 Design

32 We consider two cases:

I b d and A_s, unknown; M_d known; Since design failure is triggered by $f_s = f_y$

$$\left.\begin{array}{rrcl} \Sigma F_x = 0 & a &=& \frac{A_s f_y}{0.85 f'_c b} \\ & \rho &=& \frac{A_s}{bd} \end{array}\right\} \left.\begin{array}{rcl} a &=& \frac{\rho f_y}{0.85 f'_c} \\ M_d &=& A_s f_y \left(d - \frac{a}{2}\right) \end{array}\right\} \boxed{M_d = \Phi \underbrace{\rho f_y \left(1 - .59\rho\frac{f_y}{f'_c}\right)}_{R} bd^2} \tag{2.33-a}$$

where ρ is specified by the designer; or

$$\boxed{R = \rho f_y \left(1 - .59\rho\frac{f_y}{f'_c}\right)} \tag{2.34}$$

2.4.2 Beam Sizes, Bar Spacing, Concrete Cover

35 Beam sizes should be dimensioned as

1. Use whole inches for overall dimensions, except for slabs use $\frac{1}{2}$ inch increment.
2. Ideally, the overall depth to width ratio should be between 1.5 to 2.0 (most economical).
3. For T beams, flange thickness should be about 20% of overall depth.

36 Reinforcing bars

1. Minimum spacing between bars, and minimum covers are needed to
   (a) Prevent Honeycombing of concrete (air pockets)
   (b) Concrete (usually up to 3/4 in MSA) must pass through the reinforcement
   (c) Protect reinforcement against corrosion and fire
2. Use at least 2 bars for flexural reinforcement
3. Use bars #11 or smaller for beams.
4. Use no more than two bar sizes and no more than 2 standard sizes apart (i.e #7 and #9 acceptable; #7 and #8 or #7 and #10 not).
5. Use no more than 5 or 6 bars in one layer.
6. Place longest bars in the layer nearest to face of beam.
7. Clear distance between parallel bars not less that d_b (to avoid splitting cracks) nor 1 in. (to allow concrete to pass through).
8. Clear distance between longitudinal bars in columns not less that $1.5d_b$ or 1.5 in.
9. Minimum cover of 1.5 in.
10. Summaries in Fig. 2.7 and Table 2.1, 2.2.

2.4.3 Design Aids

37 Basic equations developed in this section can be easily graphed.

Review Given b d and known steel ratio ρ and material strength, ϕM_n can be readily obtained from $\phi M_n = \phi Rbd^2$

Design in this case

1. Set $M_d = \phi Rbd^2$
2. From tabulated values, select ρ_{max} and ρ_{min} often $0.5\rho_b$ is a good economical choice.
3. Select R from tabulated values of R in terms of f_y, f'_c and ρ. Solve for bd^2.
4. Select b and d to meet requirements. Usually depth is about 2 to 3 times the width.
5. Using tabulated values select the size and number of bars giving preference to larger bar sizes to reduce placement cost (careful about crack width!).
6. Check from tables that the selected beam width will provide room for the bars chosen with adequate cover and spacing.

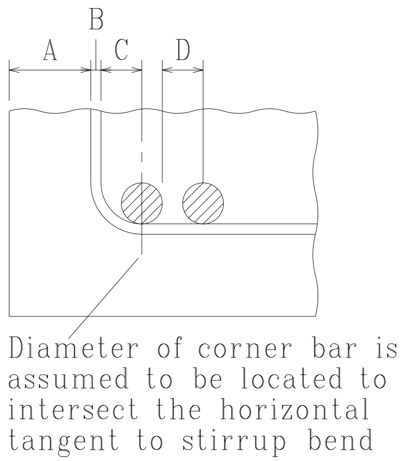
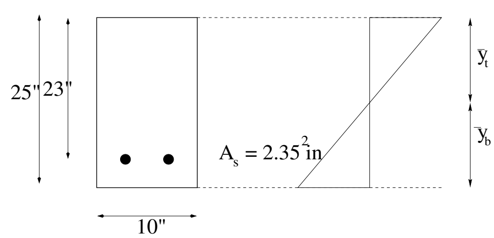


Figure 2.7: Bar Spacing

2.5 USD Examples

■ Example 2-5: Ultimate Strength; Review

Determine the ultimate moment capacity of example 2.1 $f'_c = 4{,}000$ psi; $f'_t = 475$ psi; $f_y =$ 60,000 psi; $A_s = 2.35$ in^2



Solution:

$$\rho_{act} = \frac{A_s}{bd} = \frac{2.35}{(10)(23)} = .0102 \quad (2.39\text{-a})$$

$$\rho_b = .85\beta_1 \frac{f'_c}{f_y}\frac{87}{87+f_y} = (.85)(.85)\frac{4}{60}\frac{87}{87+60} = .0285 > \rho_{act}\surd \quad (2.39\text{-b})$$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{(2.35)(60)}{(.85)(4)(10)} = 4.15 \text{ in} \quad (2.39\text{-c})$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = (2.35)(60)\left(23 - \frac{4.15}{2}\right) = 2,950 \text{ k.in} \quad (2.39\text{-d})$$

$$M_d = \phi M_n = 0.9(2,950) = \boxed{2,660 \text{ k.in}} \quad (2.39\text{-e})$$

Note:

■ **Example 2-7: Ultimate Strength; Design II**

Design a R/C beam for $b = 11.5$ in; $d = 20$ in; $f'_c = 3$ ksi; $f_y = 40$ ksi; $M_d = 1,600$ k.in
Solution:

Assume $a = \frac{d}{5} = \frac{20}{5} = 4$ in

$$A_s = \frac{M_d}{\phi f_y (d - \frac{a}{2})} = \frac{(1,600)}{(.9)(40)(20 - \frac{4}{2})} = 2.47 \text{ in}^2 \tag{2.42}$$

check assumption,

$$a = \frac{A_s f_y}{(.85) f'_c b} = \frac{(2.47)(40)}{(.85)(3)(11.5)} = 3.38 \text{ in} \tag{2.43}$$

Thus take $a = 3.3$ in.

$$A_s = \frac{(1,600)}{(.9)(40)(20 - \frac{3.3}{2})} = \boxed{2.42 \text{ in}^2} \tag{2.44-a}$$

$$\Rightarrow a = \frac{(2.42)(40)}{(.85)(3)(11.5)} = 3.3 \text{ in}\surd \tag{2.44-b}$$

$$\rho_{act} = \frac{2.42}{(11.5)(20)} = .011 \tag{2.44-c}$$

$$\rho_b = (.85)(.85)\frac{3}{40}\frac{87}{87 + 40} = .037 \tag{2.44-d}$$

$$\rho_{max} = .75\rho_b = .0278 > \rho_{act}\surd \tag{2.44-e}$$

■

■ **Example 2-8: Exact Analysis**

As an Engineer questioning the validity of the ACI equation for the ultimate flexural capacity of R/C beams, you determined experimentally the following stress strain curve for concrete:

$$\sigma = \frac{2\frac{f'_c}{\varepsilon_{max}}\varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_{max}}\right)^2} \tag{2.45}$$

where f'_c corresponds to ε_{max}.

1. Determine the exact balanced steel ratio for a R/C beam with $b = 10$", $d = 23$", $f'_c = 4,000$ psi, $f_y = 60$ ksi, $\varepsilon_{max} = 0.003$.
   (a) Determine the equation for the exact stress distribution on the section.
   (b) Determine the total compressive force C, and its location, in terms of the location of the neutral axis c.

Chapter 3

SHEAR

3.1 Introduction

1 Beams are subjected to both flexural and shear stresses. Resulting principal stresses (or stress trajectory) are shown in Fig. 3.1.

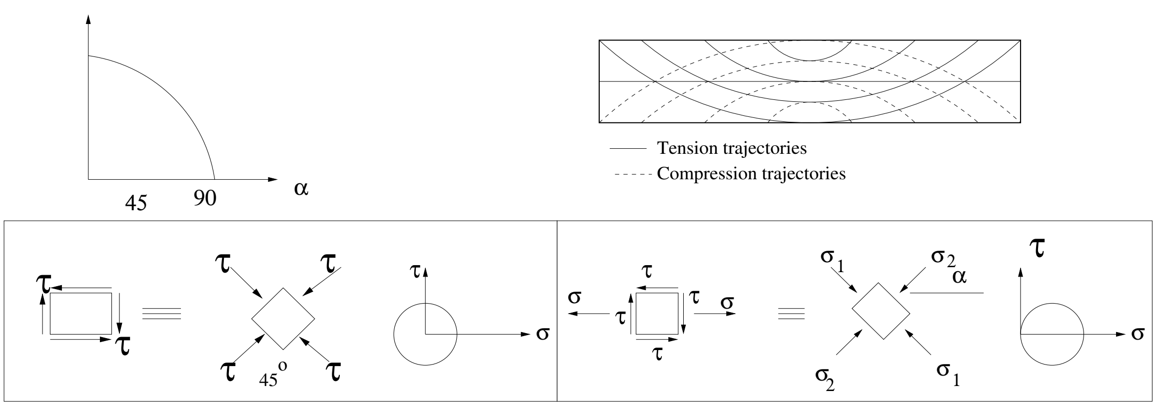


Figure 3.1: Principal Stresses in Beam

2 Due to flexure, vertical flexural cracks develop from the bottom fibers.

3 As a result of the tensile principal stresses, two types of shear cracks may develop, Fig. 3.2:

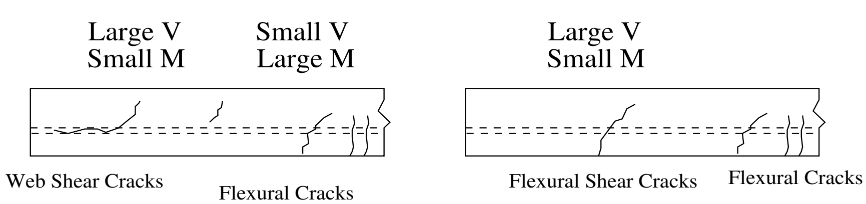


Figure 3.2: Types of Shear Cracks

Web shear cracks: Large V, small M. They initiate in the web & spread up & down at $\approx 45^o$.

3. Compute the principal stresses
4. Equate principal tensile stress to the tensile strength

10 Using a semi-analytical approach

1. Assume that f_c is directly proportional to steel stress

$$\left.\begin{array}{rcl} f_c & = & \alpha\frac{f_s}{n} \\ M_n & = & A_s f_s jd \Rightarrow f_s = \frac{M_n}{A_s jd} \end{array}\right\} \left.\begin{array}{rcl} f_c & = & \alpha\frac{M_n}{nA_s jd} \\ \rho & = & \frac{A_s}{bd} \end{array}\right\} f_c = \frac{\alpha M_n}{n\rho jbd^2} = F_1\frac{M_n}{\rho nbd^2} \tag{3.1}$$

2. Shear stress

$$v_n = F_2\frac{V_n}{bd} \tag{3.2}$$

3. From Mohr's circle, the tensile principal stress is

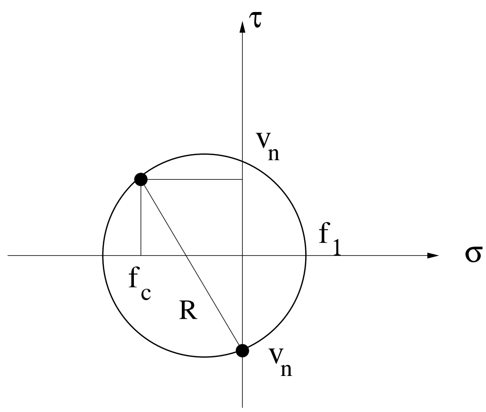


Figure 3.4: Mohr's Circle for Shear Strength of Uncracked Section

$$f_1 = \frac{f_c}{2} + \sqrt{\left(\frac{f_c}{2}\right)^2 + v_n^2} \tag{3.3}$$

4. Set f_1 equal to the tensile strength

$$f_1 = f'_t \Rightarrow f_1\frac{V_n}{bd} = f'_t\frac{V_n}{bd} \tag{3.4-a}$$

$$\frac{V_n}{bd} = \frac{f'_t}{f_1}\frac{V_n}{bd} \tag{3.4-b}$$

$$= \frac{f'_t}{\frac{f_1 bd}{V_n}} \tag{3.4-c}$$

Combining Eq. 3.1, 3.2, and 3.3

$$\frac{V_n}{bd} = \frac{f'_t}{\underbrace{\frac{F_1}{2}\frac{E_c}{E_s}}_{C'_1}\frac{M_n}{\rho V_n d} + \left[\left(\underbrace{\frac{F_1}{2}\frac{E_c}{E_s}}_{C'_1}\frac{M_n}{\rho V_n d}\right)^2 + \underbrace{F_2^2}_{C'_2}\right]^{1/2}} \tag{3.5}$$