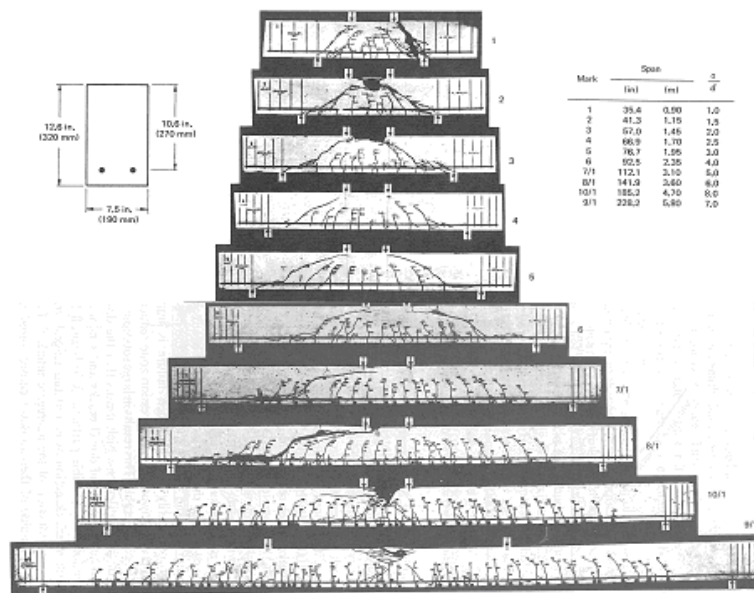


Draft

DRAFT

Lecture Notes in:  
**Mechanics and Design of  
REINFORCED CONCRETE**



**Victor E. Saouma**

Dept. of Civil Environmental and Architectural Engineering

University of Colorado, Boulder, CO 80309-0428

## Contents

<b>1</b>	<b>INTRODUCTION</b>	<b>13</b>
1.1	Material	13
1.1.1	Concrete	13
1.1.1.1	Mix Design	13
1.1.1.1.1	Constituents	13
1.1.1.1.2	Preliminary Considerations	17
1.1.1.1.3	Mix procedure	17
1.1.1.1.4	Mix Design Example	20
1.1.1.2	Mechanical Properties	22
1.1.2	Reinforcing Steel	26
1.2	Design Philosophy, USD	27
1.3	Analysis vs Design	28
1.4	Basic Relations and Assumptions	28
1.5	ACI Code	29
<b>2</b>	<b>FLEXURE</b>	<b>31</b>
2.1	Uncracked Section	31
E 2-1	Uncracked Section	32
2.2	Section Cracked, Stresses Elastic	33
2.2.1	Basic Relations	33
2.2.2	Working Stress Method	34
E 2-2	Cracked Elastic Section	35
E 2-3	Working Stress Design Method; Analysis	36
E 2-4	Working Stress Design Method; Design	37
2.3	Cracked Section, Ultimate Strength Design Method	38
2.3.1	Whitney Stress Block	38
2.3.2	Balanced Design	40
2.3.3	Review	41
2.3.4	Design	41
2.4	Practical Design Considerations	42
2.4.1	Minimum Depth	42
2.4.2	Beam Sizes, Bar Spacing, Concrete Cover	43
2.4.3	Design Aids	43
2.5	USD Examples	45
E 2-5	Ultimate Strength; Review	45
E 2-6	Ultimate Strength; Design I	46
E 2-7	Ultimate Strength; Design II	47

<b>6</b>	<b>SERVICEABILITY</b>	<b>103</b>
6.1	Control of Cracking . . . . .	103
	E 6-1 Crack Width . . . . .	105
6.2	Deflections . . . . .	105
	6.2.1 Short Term Deflection . . . . .	106
	6.2.2 Long Term Deflection . . . . .	107
	E 6-2 Deflections . . . . .	109
<b>7</b>	<b>APPROXIMATE FRAME ANALYSIS</b>	<b>111</b>
7.1	Vertical Loads . . . . .	111
7.2	Horizontal Loads . . . . .	114
	7.2.1 Portal Method . . . . .	114
	E 7-1 Approximate Analysis of a Frame subjected to Vertical and Horizontal Loads	116
<b>8</b>	<b>COLUMNS</b>	<b>131</b>
<b>9</b>	<b>COLUMNS</b>	<b>133</b>
9.1	Introduction . . . . .	133
	9.1.1 Types of Columns . . . . .	133
	9.1.2 Possible Arrangement of Bars . . . . .	134
9.2	Short Columns . . . . .	134
	9.2.1 Concentric Loading . . . . .	134
	9.2.2 Eccentric Columns . . . . .	134
	9.2.2.1 Balanced Condition . . . . .	135
	9.2.2.2 Tension Failure . . . . .	137
	9.2.2.3 Compression Failure . . . . .	138
	9.2.3 ACI Provisions . . . . .	139
	9.2.4 Interaction Diagrams . . . . .	139
	9.2.5 Design Charts . . . . .	139
	E 9-1 R/C Column, $c$ known . . . . .	139
	E 9-2 R/C Column, $e$ known . . . . .	141
	E 9-3 R/C Column, Using Design Charts . . . . .	145
	9.2.6 Biaxial Bending . . . . .	146
	E 9-4 Biaxially Loaded Column . . . . .	149
9.3	Long Columns . . . . .	150
	9.3.1 Euler Elastic Buckling . . . . .	150
	9.3.2 Effective Length . . . . .	151
	9.3.3 Moment Magnification Factor; ACI Provisions . . . . .	153
	E 9-5 Long R/C Column . . . . .	155
	E 9-6 Design of Slender Column . . . . .	157
<b>10</b>	<b>PRESTRESSED CONCRETE</b>	<b>159</b>
10.1	Introduction . . . . .	159
	10.1.1 Materials . . . . .	159
	10.1.2 Prestressing Forces . . . . .	162
	10.1.3 Assumptions . . . . .	162
	10.1.4 Tendon Configuration . . . . .	162
	10.1.5 Equivalent Load . . . . .	162

## List of Figures

1.1	Schematic Representation of Aggregate Gradation	14
1.2	MicroCracks in Concrete under Compression	23
1.3	Concrete Stress Strain Curve	23
1.4	Modulus of Rupture Test	24
1.5	Split Cylinder (Brazilian) Test	24
1.6	Biaxial Strength of Concrete	25
1.7	Time Dependent Strains in Concrete	26
2.1	Strain Diagram Uncracked Section	31
2.2	Transformed Section	32
2.3	Stress Diagram Cracked Elastic Section	33
2.4	Desired Stress Distribution; WSD Method	34
2.5	Cracked Section, Limit State	39
2.6	Whitney Stress Block	40
2.7	Bar Spacing	45
2.8	T Beams	50
2.9	T Beam as Rectangular Section	50
2.10	T Beam Strain and Stress Diagram	51
2.11	Decomposition of Steel Reinforcement for T Beams	51
2.12	Doubly Reinforced Beams; Strain and Stress Diagrams	56
2.13	Different Possibilities for Doubly Reinforced Concrete Beams	57
2.14	Strain Diagram, Doubly Reinforced Beam; is $A_s$ Yielding?	57
2.15	Strain Diagram, Doubly Reinforced Beam; is $A'_s$ Yielding?	58
2.16	Summary of Conditions for top and Bottom Steel Yielding	59
2.17	Bending of a Beam	64
2.18	Moment-Curvature Relation for a Beam	64
2.19	Bond and Development Length	65
2.20	Actual Bond Distribution	67
2.21	Splitting Along Reinforcement	67
2.22	Development Length	67
2.23	Development Length	68
2.24	Hooks	69
2.25	Bar cutoff requirements of the ACI code	71
2.26	Standard cutoff or bend points for bars in approximately equal spans with uniformly distributed load	73
2.27	Moment Capacity Diagram	73
3.1	Principal Stresses in Beam	75

9.3	Possible Bar arrangements . . . . .	134
9.4	Sources of Bending . . . . .	135
9.5	Load Moment Interaction Diagram . . . . .	135
9.6	Strain and Stress Diagram of a R/C Column . . . . .	136
9.7	Column Interaction Diagram . . . . .	140
9.8	Failure Surface of a Biaxially Loaded Column . . . . .	146
9.9	Load Contour at Plane of Constant $P_n$ , and Nondimensionalized Corresponding plots . . . . .	147
9.10	Biaxial Bending Interaction Relations in terms of $\beta$ . . . . .	148
9.11	Bilinear Approximation for Load Contour Design of Biaxially Loaded Columns . . . . .	148
9.12	Euler Column . . . . .	150
9.13	Column Failures . . . . .	151
9.14	Critical lengths of columns . . . . .	152
9.15	Effective length Factors $\Psi$ . . . . .	153
9.16	Standard Alignment Chart (ACI) . . . . .	154
9.17	Minimum Column Eccentricity . . . . .	154
9.18	P-M Magnification Interaction Diagram . . . . .	155
10.1	Pretensioned Prestressed Concrete Beam, (?) . . . . .	160
10.2	Posttensioned Prestressed Concrete Beam, (?) . . . . .	160
10.3	7 Wire Prestressing Tendon . . . . .	161
10.4	Alternative Schemes for Prestressing a Rectangular Concrete Beam, (?) . . . . .	163
10.5	Determination of Equivalent Loads . . . . .	163
10.6	Load-Deflection Curve and Corresponding Internal Flexural Stresses for a Typical Prestressed Concrete . . . . .	
10.7	Flexural Stress Distribution for a Beam with Variable Eccentricity; Maximum Moment Section and Support . . . . .	
10.8	Walnut Lane Bridge, Plan View . . . . .	169
10.9	Walnut Lane Bridge, Cross Section . . . . .	170

## List of Tables

1.1	ASTM Sieve Designation's Nominal Sizes Used for Concrete Aggregates . . . . .	15
1.2	ASTM C33 Grading Limits for Coarse Concrete Aggregates . . . . .	15
1.3	ASTM C33 Grading Limits for Fine Concrete Aggregates . . . . .	15
1.4	Example of Fineness Modulus Determination for Fine Aggregate . . . . .	17
1.5	Recommended Slumps (inches) for Various Types of Construction . . . . .	18
1.6	Recommended Average Total Air Content as % of Different Nominal Maximum Sizes of Aggregates and	
1.7	Approximate Mixing Water Requirements, lb/yd <sup>3</sup> of Concrete For Different Slumps and Nominal Maxim	
1.8	Relationship Between Water/Cement Ratio and Compressive Strength . . . . .	19
1.9	Volume of Dry-Rodded Coarse Aggregate per Unit Volume of Concrete for Different Fineness Moduli of	
1.10	Creep Coefficients . . . . .	25
1.11	Properties of Reinforcing Bars . . . . .	26
1.12	Strength Reduction Factors, $\Phi$ . . . . .	27
2.1	Total areas for various numbers of reinforcing bars (inch <sup>2</sup> ) . . . . .	44
2.2	Minimum Width (inches) according to ACI Code . . . . .	44
4.1	Building Structural Systems . . . . .	93
5.1	Recommended Minimum Slab and Beam Depths . . . . .	98
7.1	Columns Combined Approximate Vertical and Horizontal Loads . . . . .	128
7.2	Girders Combined Approximate Vertical and Horizontal Loads . . . . .	129

## Chapter 1

# INTRODUCTION

### 1.1 Material

#### 1.1.1 Concrete

This section is adapted from *Concrete* by Mindess and Young, Prentice Hall, 1981

##### 1.1.1.1 Mix Design

###### 1.1.1.1.1 Constituents

<sup>1</sup> Concrete is a mixture of Portland cement, water, and aggregates (usually sand and crushed stone).

<sup>2</sup> Portland cement is a mixture of calcareous and argillaceous materials which are calcined in a kiln and then pulverized. When mixed with water, cement hardens through a process called **hydration**.

<sup>3</sup> Ideal mixture is one in which:

1. A minimum amount of cement-water paste is used to fill the interstices between the particles of aggregates.
2. A minimum amount of water is provided to complete the chemical reaction with cement. Strictly speaking, a water/cement ratio of about 0.25 is needed to complete this reaction, but then the concrete will have a very low “workability”.

In such a mixture, about 3/4 of the volume is constituted by the aggregates, and the remaining 1/4 being the cement paste.

<sup>4</sup> Smaller particles up to 1/4 in. in size are called **fine aggregates**, and the larger ones being **coarse aggregates**.

<sup>5</sup> Portland Cement has the following ASTM designation

I Normal

II Moderate sulfate resistant, moderate heat of hydration

III High early strength (but releases too much heat)

ASTM Design.	Size	
	mm	in.
Coarse Aggregate		
3 in.	75	3
2 <sup>1/2</sup> in.	63	2.5
2 in.	50	2
1 <sup>1/2</sup> in.	37.5	1.5
1 in.	25	1
3/4 in.	19	0.75
1/2 in.	12.5	0.50
3/8 in.	9.5	0.375
Fine Aggregate		
No. 4	4.75	0.187
No. 8	2.36	0.0937
No. 16	1.18	0.0469
No. 30	0.60 (600 μm)	0.0234
No. 50	300 μm	0.0124
No. 100	150 μm	0.0059

Table 1.1: ASTM Sieve Designation's Nominal Sizes Used for Concrete Aggregates

Sieve Size	% Passing Each Sieve (Nominal Maximum Size)			
	1 <sup>1/2</sup> in.	1 in.	3/4 in.	1/2 in.
1 <sup>1/2</sup> in.	95-100	100	-	-
1 in.	-	95-100	100	-
3/4 in.	35-70	-	90-100	100
1/2 in.	-	25-60	-	90-100
3/8 in.	10-30	-	20-55	40-70
No. 4	0-5	0-10	0-10	0-15
No. 8	-	0-5	0-5	0-5

Table 1.2: ASTM C33 Grading Limits for Coarse Concrete Aggregates

Sieve Size	% Passing
3/4 in.	100
No. 4	95-100
No. 8	80-100
No. 16	50-85
No. 30	25-60
No. 50	10-30
No. 100	2-10

Table 1.3: ASTM C33 Grading Limits for Fine Concrete Aggregates

Sieve Size	Weight Retained (g)	Amount Retained (wt. %)	Cumulative Amount Retained (%)	Cumulative Amount Passing (%)
No. 4	9	2	2	98
No. 8	46	9	11	89
No. 16	97	19	30	70
No. 30	99	20	50	50
No. 50	120	24	74	26
No. 100	91	18	92	8
Sample Weight 500 g.			$\Sigma = 259$	
Fineness modulus= $259/100=2.59$				

Table 1.4: Example of Fineness Modulus Determination for Fine Aggregate

### 1.1.1.1.2 Preliminary Considerations

24 There are two fundamental aspects to mix design to keep in mind:

1. **Water/Cement ratio:** where the strength is inversely proportional to the water to cement ratio, approximately expressed as:

$$f'_c = \frac{A}{B^{1.5w/c}} \quad (1.4)$$

For  $f'_c$  in psi,  $A$  is usually taken as 14,000 and  $B$  depends on the type of cement, but may be taken to be about 4. It should be noted that  $w/c$  controls not only the strength, but also the **porosity** and hence the **durability**.

2. **Aggregate Grading:** In order to minimize the amount of cement paste, we must maximize the volume of aggregates. This can be achieved through proper packing of the granular material. The “ideal” grading curve (with minimum voids) is closely approximated by the Fuller curve

$$P_t = \left(\frac{d}{D}\right)^q \quad (1.5)$$

where  $P_t$  is the fraction of total solids finer than size  $d$ , and  $D$  is the maximum particle size,  $q$  is generally taken as  $1/2$ , hence the *parabolic* grading.

### 1.1.1.1.3 Mix procedure

25 Before starting the mix design process, the following **material properties** should be determined:

1. Sieve analysis of both fine and coarse aggregates
2. Unit weight of the coarse aggregate
3. Bulk specific gravities
4. absorption capacities of the aggregates

Slump in.	Sizes of Aggregates				
	3/8 in.	1/2 in.	3/4 in.	1 in.	1 <sup>1/2</sup> in.
Non-Air-Entrained Concrete					
1-2	350	335	315	300	275
3-4	385	365	340	325	300
6-7	410	385	360	340	315
Air-Entrained Concrete					
1-2	305	295	280	270	250
3-4	340	325	305	295	275
6-7	365	345	325	310	290

Table 1.7: Approximate Mixing Water Requirements, lb/yd<sup>3</sup> of Concrete For Different Slumps and Nominal Maximum Sizes of Aggregates

28 days $f'_c$	$w/c$ Ratio by Weight	
	Non-air-entrained	Air-entrained
6,000	0.41	-
5,000	0.48	0.40
4,000	0.57	0.48
3,000	0.68	0.59
2,000	0.82	0.74

Table 1.8: Relationship Between Water/Cement Ratio and Compressive Strength

**Fine Aggregates:** Bulk specific gravity (SSD) = 2.65; absorption capacity = 1.3 %; Total moisture content=5.5%; fineness modulus = 2.70

The sieve analyses of both the coarse and fine aggregates fall within the specified limits. With this information, the mix design can proceed:

1. Choice of **slump** is consistent with Table 1.5.
2. **Maximum aggregate size** (3/4 in) is governed by reinforcing details.
3. Estimation of mixing **water**: Because water will be exposed to freeze and thaw, it must be air-entrained. From Table 1.6 the air content recommended for extreme exposure is 6.0%, and from Table 1.7 the water requirement is 280 lb/yd<sup>3</sup>
4. From Table 1.8, the **water to cement ratio** estimate is 0.4
5. **Cement content**, based on steps 4 and 5 is 280/0.4=700 lb/yd<sup>3</sup>
6. **Coarse aggregate content**, interpolating from Table 1.9 for the fineness modulus of the fine aggregate of 2.70, the volume of dry-rodded coarse aggregate per unit volume of concrete is 0.63. Therefore, the coarse aggregate will occupy 0.63 × 27 = 17.01 ft<sup>3</sup>/yd<sup>3</sup>. The OD weight of the coarse aggregate is 17.01 ft<sup>3</sup>/yd<sup>3</sup>, × 100 lbs/ft<sup>3</sup>=1,701 lb. The SSD weight is 1,701 × 1.01=1,718 lb.
7. **Fine aggregate content** Knowing the weights and specific gravities of the water, cement, and coarse aggregate, and knowing the air volume, we can calculate the volume per yd<sup>3</sup> occupied by the different ingredients.

Water	280/62.4	=	4.49	ft <sup>3</sup>
Cement	700/(3.15)(62.4)	=	3.56	ft <sup>3</sup>
Coarse Aggregate (SSD)	1,718/(2.70)(62.4)	=	1.62	ft <sup>3</sup>
Air	(0.06)(27)	=	1.62	ft <sup>3</sup>
			19.87	ft <sup>3</sup>

Hence, the fine aggregate must occupy a volume of 27.0 – 19.87 = 7.13 ft<sup>3</sup>. The required SSD weight of the fine aggregate is 7.13 ft<sup>3</sup> (2.65)(62.4)lb/ft<sup>3</sup> =1,179 lbs lb.

8. **Adjustment for moisture** in the aggregate. Since the aggregate will be neither SSD or OD in the field, it is necessary to adjust the aggregate weights for the amount of water contained in the aggregate. Only surface water need be considered; absorbed water does not become part of the mix water. For the given moisture contents, the adjusted aggregate weights become:

Coarse aggregate (wet)=1,718(1.025-0.01) = 1,744 lb/yd<sup>3</sup> of dry coarse  
 Fine aggregate (wet)=1,179(1.055-0.013) = 1,229 lb/yd<sup>3</sup> of dry fine

Surface moisture contributed by the coarse aggregate is 2.5-1.0 = 1.5%; by the fine aggregate: 5.5-1.3 = 4.2%; Hence we need to decrease water to 280-1,718(0.015)-1,179(0.042) = 205 lb/yd<sup>3</sup>.

Thus, the estimated batch weight per yd<sup>3</sup> are

## Chapter 2

# FLEXURE

<sup>1</sup> This is probably the longest chapter in the notes, we shall cover in great details flexural design/analysis of R/C beams starting with uncracked section to failure conditions.

1. Uncracked elastic (uneconomical)
2. cracked elastic (service stage)
3. Ultimate (failure)

### 2.1 Uncracked Section

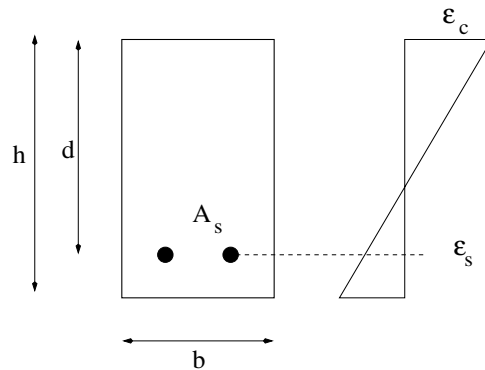


Figure 2.1: Strain Diagram Uncracked Section

<sup>2</sup> Assuming perfect bond between steel and concrete, we have  $\epsilon_s = \epsilon_c$ , Fig. 2.1

$$\epsilon_s = \epsilon_c \Rightarrow \frac{f_s}{E_s} = \frac{f_c}{E_c} \Rightarrow f_s = \frac{E_s}{E_c} f_c \Rightarrow f_s = n f_c \quad (2.1)$$

where  $n$  is the modular ratio  $n = \frac{E_s}{E_c}$

<sup>3</sup> Tensile force in steel  $T_s = A_s f_s = A_s n f_c$

<sup>4</sup> Replace steel by an equivalent area of concrete, Fig. 2.2.

$$f_{ct} = \frac{Mc}{I} = \frac{(540,000) \text{ lb.in}(25 - 13.2) \text{ in}}{(14,722) \text{ in}^4} = \boxed{433 \text{ psi}} < 475 \text{ psi} \checkmark \quad (2.3-h)$$

$$f_s = n \frac{Mc}{I} = (8) \frac{(540,000)(23 - 13.2) \text{ in}}{(14,722)} = \boxed{2,876 \text{ psi}} \quad (2.3-i)$$

■

## 2.2 Section Cracked, Stresses Elastic

7 This is important not only as an acceptable alternative ACI design method, but also for the later evaluation of crack width under service loads.

### 2.2.1 Basic Relations

8 If  $f_{ct} > f_r$ ,  $f_{cc} < \approx .5f'_c$  and  $f_s < f_y$  we will assume that the crack goes all the way to the N.A and we will use the transformed section, Fig. 2.3

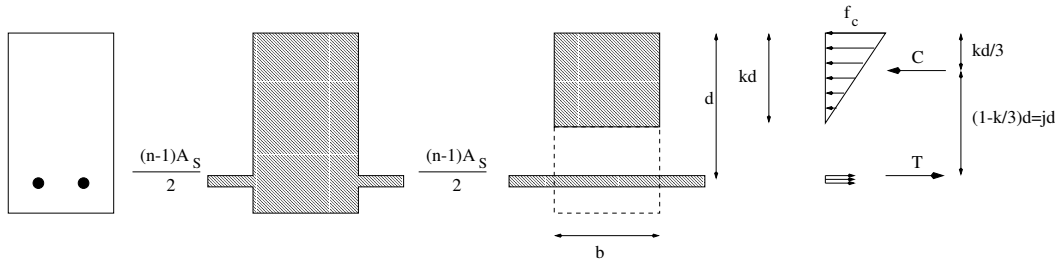


Figure 2.3: Stress Diagram Cracked Elastic Section

9 To locate N.A, tension force = compressive force (by def. NA) (Note, for linear stress distribution and with  $\Sigma F_x = 0$ ;  $\sigma = by \Rightarrow \int bydA = 0$ , thus  $\int bydA = 0$  and  $\int ydA = \bar{y}A = 0$ , by definition, gives the location of the neutral axis)

10 Note, N.A. location depends only on geometry &  $n \left( \frac{E_s}{E_c} \right)$

11 Tensile and compressive forces are equal to  $C = \frac{bkd}{2} f_c$  &  $T = A_s f_s$  and neutral axis is determined by equating the moment of the tension area to the moment of the compression area

$$b(kd) \left( \frac{kd}{2} \right) = nA_s(d - kd) \quad 2^{nd} \text{ degree equation} \quad (2.4-a)$$

$$M = Tjd = A_s f_s jd \Rightarrow f_s = \frac{M}{A_s jd} \quad (2.4-b)$$

$$M = Cjd = \frac{bkd}{2} f_c jd = \frac{bd^2}{2} kj f_c \Rightarrow \boxed{f_c = \frac{M}{\frac{1}{2}bd^2kj}} \quad (2.4-c)$$

where  $j = (1 - k/3)$ .

**Review** Start by determining  $\rho$ ,

- If  $\rho < \rho_b$  steel reaches max. allowable value before concrete, and

$$M = A_s f_s j d \tag{2.9}$$

- If  $\rho > \rho_b$  concrete reaches max. allowable value before steel and

$$M = f_c \frac{b k d}{2} j d \tag{2.10}$$

or

$$M = \frac{1}{2} f_c j k b d^2 = R b d^2 \tag{2.11}$$

where

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

**Design** We define

$$R \stackrel{\text{def}}{=} \frac{1}{2} f_c k j \tag{2.12}$$

where  $k = \frac{n}{n+r}$ , solve for  $b d^2$  from

$$b d^2 = \frac{M}{R} \tag{2.13}$$

assume  $b$  and solve for  $d$ . Finally we can determine  $A_s$  from

$$A_s = \rho_b b d \tag{2.14}$$

17 Summary

Review	Design
$b, d, A_s \checkmark$ $M?$	$M \checkmark$ $b, d, A_s?$
$\rho = \frac{A_s}{b d}$	$k = \frac{n}{n+r}$
$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$	$j = 1 - \frac{k}{3}$
$r = \frac{f_s}{f_c}$	$R = \frac{1}{2} f_c k j$
$\rho_b = \frac{n}{2r(n+r)}$	$\rho_b = \frac{n}{2r(n+r)}$
$\rho < \rho_b \quad M = A_s f_s j d$	$b d^2 = \frac{M}{R}$
$\rho > \rho_b \quad M = \frac{1}{2} f_c b k d^2 j$	$A_s = \rho_b b d$ or $A_s = \frac{M}{f_s j d}$

■ Example 2-2: Cracked Elastic Section

Solution:

$$\rho = \frac{A_s}{bd} = \frac{2.35}{(10)(23)} = .0102 \quad (2.16-a)$$

$$f_s = 24 \text{ ksi} \quad (2.16-b)$$

$$f_c = (.45)(4,000) = 1,800 \text{ psi} \quad (2.16-c)$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(.0102)8 + (.0102)^2} - (8)(.0102) = .331 \quad (2.16-d)$$

$$j = 1 - \frac{k}{3} = .889 \quad (2.16-e)$$

$$N.A. \text{ @ } (.331)(23) = 7.61 \text{ in} \quad (2.16-f)$$

$$\rho_b = \frac{n}{2r(n+r)} = \frac{8}{(2)(13.33)(8+13.33)} = .014 > \rho \Rightarrow \text{Steel reaches elastic limit} \quad (2.16-g)$$

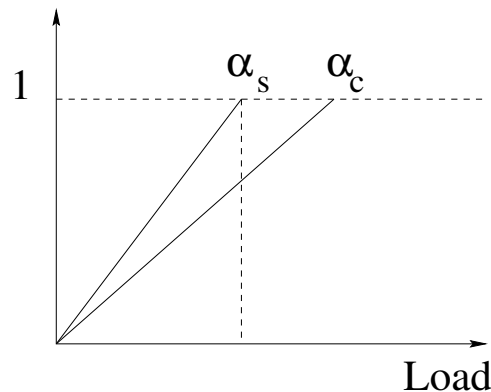
$$M = A_s f_s j d = (2.35)(24)(.889)(23) = \boxed{1,154 \text{ k.in} = 96 \text{ k.ft}} \quad (2.16-h)$$

Note, had we used the alternate equation for moment (wrong) we would have overestimated the design moment:

$$M = \frac{1}{2} f_c b k d^2 j \quad (2.17-a)$$

$$= \frac{1}{2} (1.8)(10)(0.33)(0.89)(23)^2 = 1,397 \text{ k.in} > 1,154 \text{ k.in} \quad (2.17-b)$$

If we define  $\alpha_c = f_c/1,800$  and  $\alpha_s = f_s/24,000$ , then as the load increases both  $\alpha_c$  and  $\alpha_s$  increase, but at different rates, one of them  $\alpha_s$  reaches 1 before the other.



■

■ **Example 2-4: Working Stress Design Method; Design**

Design a beam to carry  $LL = 1.9 \text{ k/ft}$ ,  $DL = 1.0 \text{ k/ft}$  with  $f'_c = 4,000 \text{ psi}$ ,  $f_y = 60,000 \text{ psi}$ ,  $L = 32 \text{ ft}$ .

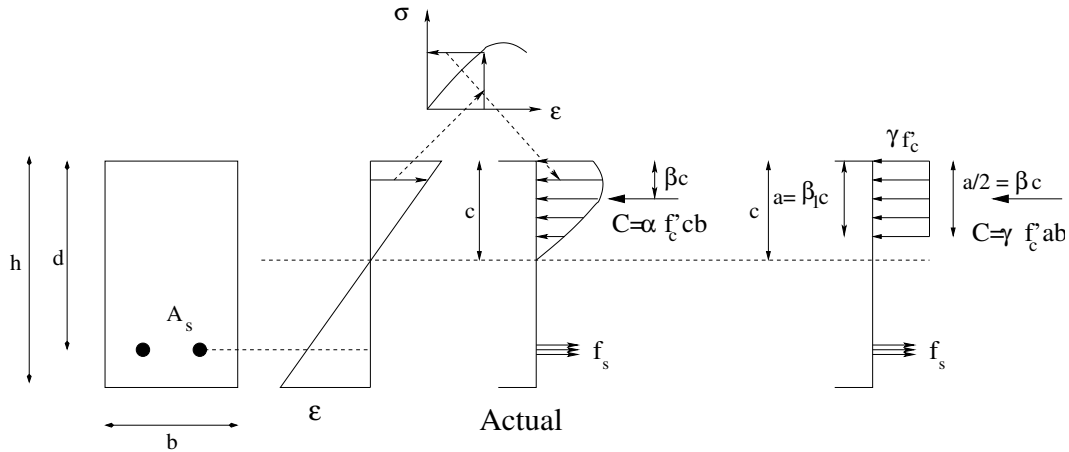


Figure 2.5: Cracked Section, Limit State

$$\alpha = \frac{f_{av}}{f'_c} \tag{2.20-b}$$

$$a = \beta_1 c \tag{2.20-c}$$

Thus

$$\gamma = \frac{\alpha}{\beta_1} \tag{2.21}$$

But the location of the resultant forces must be the same, hence

$$\beta_1 = 2\beta \tag{2.22}$$

21 From Experiments

$f'_c$ (psi)	<4,000	5,000	6,000	7,000	8,000
$\alpha$	.72	.68	.64	.60	.56
$\beta$	.425	.400	.375	.350	.325
$\beta_1 = 2\beta$	.85	.80	.75	.70	.65
$\gamma = \alpha/\beta_1$	0.85	0.85	0.85	0.86	0.86

22 Thus we have, (**ACI-318 10.2.7.3**):

$$\beta_1 = \begin{cases} .85 & \text{if } f'_c \leq 4,000 \\ .85 - (.05)(f'_c - 4,000) \frac{1}{1,000} & \text{if } 4,000 < f'_c < 8,000 \end{cases} \tag{2.23}$$

23 Failure can occur by either

**yielding of steel:**  $\epsilon_s = \epsilon_y$ ; Progressive

**crushing of concrete:**  $\epsilon_c = .003$ ; Sudden; (**ACI 10.3.2**).

27 Also we need to specify a minimum reinforcement ratio

$$\rho_{min} \geq \frac{200}{f_y} \quad (\text{ACI 10.5.1}) \quad (2.29)$$

to account for temperature & shrinkage

28 Note, that  $\rho$  need not be as high as  $0.75\rho_b$ . If steel is relatively expensive, or deflection is of concern, can use lower  $\rho$ .

29 As a rule of thumb, if  $\rho < 0.5\rho_b$ , there is no need to check for deflection.

### 2.3.3 Review

30 Given,  $b, d, A_s, f'_c, f_y$ , determine the moment capacity  $M$ .

$$\begin{aligned} \rho_{act} &= \frac{A_s}{bd} \\ \rho_b &= (.85)\beta_1 \frac{f'_c}{f_y} \frac{87}{87+f_y} \end{aligned} \quad (2.30)$$

- $\rho_{act} < \rho_b$ : Failure by yielding and

$$\begin{aligned} a &= \frac{A_s f_y}{.85 f'_c b} & \Sigma F_x &= 0 \\ M_d &= \phi A_s f_y (d - \frac{a}{2}) & \Sigma M &= 0 \end{aligned} \quad (2.31)$$

- $\rho_{act} > \rho_b$  is not allowed by code, in this case we have an extra unknown  $f_s$ .

31 We now have one more unknown  $f_s$ , and we will need an additional equation (from strain diagram).

$$\begin{aligned} c &= \frac{A_s f_s}{.85 f'_c b \beta_1} & \Sigma F_x &= 0 \\ \frac{c}{d} &= \frac{.003}{.003 + \epsilon_s} & \text{From strain diagram} & \\ M_d &= \phi A_s f_s (d - \frac{\beta_1 c}{2}) & \Sigma M &= 0 \end{aligned} \quad (2.32)$$

We can solve by iteration, or substitution and solution of a quadratic equation.

### 2.3.4 Design

32 We consider two cases:

I  $b, d$  and  $A_s$ , unknown;  $M_d$  known; Since design failure is triggered by  $f_s = f_y$

$$\left. \begin{aligned} \Sigma F_x = 0 \quad a &= \frac{A_s f_y}{.85 f'_c b} \\ \rho &= \frac{A_s}{bd} \end{aligned} \right\} \left. \begin{aligned} a &= \frac{\rho f_y}{.85 f'_c} \\ M_d &= A_s f_y (d - \frac{a}{2}) \end{aligned} \right\} M_d = \underbrace{\Phi \rho f_y \left(1 - .59 \rho \frac{f_y}{f'_c}\right)}_R \quad (2.33-a)$$

where  $\rho$  is specified by the designer; or

$$R = \rho f_y \left(1 - .59 \rho \frac{f_y}{f'_c}\right) \quad (2.34)$$

### 2.4.2 Beam Sizes, Bar Spacing, Concrete Cover

35 Beam sizes should be dimensioned as

1. Use whole inches for overall dimensions, except for slabs use  $\frac{1}{2}$  inch increment.
2. Ideally, the overall depth to width ratio should be between 1.5 to 2.0 (most economical).
3. For T beams, flange thickness should be about 20% of overall depth.

36 Reinforcing bars

1. Minimum spacing between bars, and minimum covers are needed to
  - (a) Prevent Honeycombing of concrete (air pockets)
  - (b) Concrete (usually up to 3/4 in MSA) must pass through the reinforcement
  - (c) Protect reinforcement against corrosion and fire
2. Use at least 2 bars for flexural reinforcement
3. Use bars #11 or smaller for beams.
4. Use no more than two bar sizes and no more than 2 standard sizes apart (i.e #7 and #9 acceptable; #7 and #8 or #7 and #10 not).
5. Use no more than 5 or 6 bars in one layer.
6. Place longest bars in the layer nearest to face of beam.
7. Clear distance between parallel bars not less than  $d_b$  (to avoid splitting cracks) nor 1 in. (to allow concrete to pass through).
8. Clear distance between longitudinal bars in columns not less than  $1.5d_b$  or 1.5 in.
9. Minimum cover of 1.5 in.
10. Summaries in Fig. 2.7 and Table 2.1, 2.2.

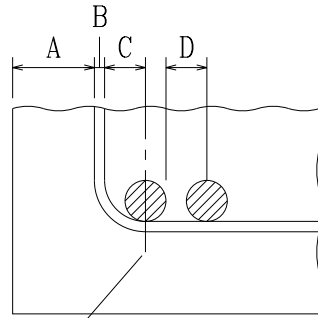
### 2.4.3 Design Aids

37 Basic equations developed in this section can be easily graphed.

**Review** Given  $b$   $d$  and known steel ratio  $\rho$  and material strength,  $\phi M_n$  can be readily obtained from  $\phi M_n = \phi Rbd^2$

**Design** in this case

1. Set  $M_d = \phi Rbd^2$
2. From tabulated values, select  $\rho_{max}$  and  $\rho_{min}$  often  $0.5\rho_b$  is a good economical choice.
3. Select  $R$  from tabulated values of  $R$  in terms of  $f_y$ ,  $f'_c$  and  $\rho$ . Solve for  $bd^2$ .
4. Select  $b$  and  $d$  to meet requirements. Usually depth is about 2 to 3 times the width.
5. Using tabulated values select the size and number of bars giving preference to larger bar sizes to reduce placement cost (careful about crack width!).
6. Check from tables that the selected beam width will provide room for the bars chosen with adequate cover and spacing.



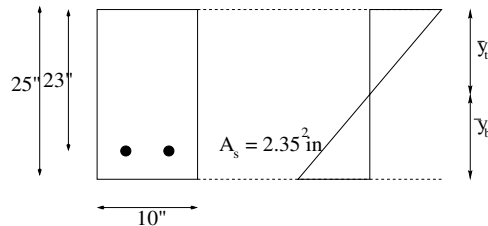
Diameter of corner bar is assumed to be located to intersect the horizontal tangent to stirrup bend

Figure 2.7: Bar Spacing

## 2.5 USD Examples

### ■ Example 2-5: Ultimate Strength; Review

Determine the ultimate moment capacity of example 2.1  $f'_c = 4,000$  psi;  $f'_t = 475$  psi;  $f_y = 60,000$  psi;  $A_s = 2.35$  in<sup>2</sup>



**Solution:**

$$\rho_{act} = \frac{A_s}{bd} = \frac{2.35}{(10)(23)} = .0102 \quad (2.39-a)$$

$$\rho_b = .85\beta_1 \frac{f'_c}{f_y} \frac{87}{87 + f_y} = (.85)(.85) \frac{4}{60} \frac{87}{87 + 60} = .0285 > \rho_{act} \quad (2.39-b)$$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{(2.35)(60)}{(.85)(4)(10)} = 4.15 \text{ in} \quad (2.39-c)$$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right) = (2.35)(60) \left( 23 - \frac{4.15}{2} \right) = 2,950 \text{ k.in} \quad (2.39-d)$$

$$M_d = \phi M_n = 0.9(2,950) = \boxed{2,660 \text{ k.in}} \quad (2.39-e)$$

Note:

### ■ Example 2-7: Ultimate Strength; Design II

Design a R/C beam for  $b = 11.5$  in;  $d = 20$  in;  $f'_c = 3$  ksi;  $f_y = 40$  ksi;  $M_d = 1,600$  k.in

**Solution:**

Assume  $a = \frac{d}{5} = \frac{20}{5} = 4$  in

$$A_s = \frac{M_d}{\phi f_y (d - \frac{a}{2})} = \frac{(1,600)}{(.9)(40)(20 - \frac{4}{2})} = 2.47 \text{ in}^2 \quad (2.42)$$

check assumption,

$$a = \frac{A_s f_y}{(.85) f'_c b} = \frac{(2.47)(40)}{(.85)(3)(11.5)} = 3.38 \text{ in} \quad (2.43)$$

Thus take  $a = 3.3$  in.

$$A_s = \frac{(1,600)}{(.9)(40)(20 - \frac{3.3}{2})} = \boxed{2.42 \text{ in}^2} \quad (2.44\text{-a})$$

$$\Rightarrow a = \frac{(2.42)(40)}{(.85)(3)(11.5)} = 3.3 \text{ in} \quad (2.44\text{-b})$$

$$\rho_{act} = \frac{2.42}{(11.5)(20)} = .011 \quad (2.44\text{-c})$$

$$\rho_b = (.85)(.85) \frac{3}{40} \frac{87}{87 + 40} = .037 \quad (2.44\text{-d})$$

$$\rho_{max} = .75 \rho_b = .0278 > \rho_{act} \quad (2.44\text{-e})$$

■

### ■ Example 2-8: Exact Analysis

As an Engineer questioning the validity of the ACI equation for the ultimate flexural capacity of R/C beams, you determined experimentally the following stress strain curve for concrete:

$$\sigma = \frac{2 \frac{f'_c}{\varepsilon_{max}} \varepsilon}{1 + \left( \frac{\varepsilon}{\varepsilon_{max}} \right)^2} \quad (2.45)$$

where  $f'_c$  corresponds to  $\varepsilon_{max}$ .

1. Determine the exact balanced steel ratio for a R/C beam with  $b = 10''$ ,  $d = 23''$ ,  $f'_c = 4,000$  psi,  $f_y = 60$  ksi,  $\varepsilon_{max} = 0.003$ .
  - (a) Determine the equation for the exact stress distribution on the section.
  - (b) Determine the total compressive force  $C$ , and its location, in terms of the location of the neutral axis  $c$ .

## Chapter 3

# SHEAR

### 3.1 Introduction

1 Beams are subjected to both flexural and shear stresses. Resulting principal stresses (or stress trajectory) are shown in Fig. 3.1.

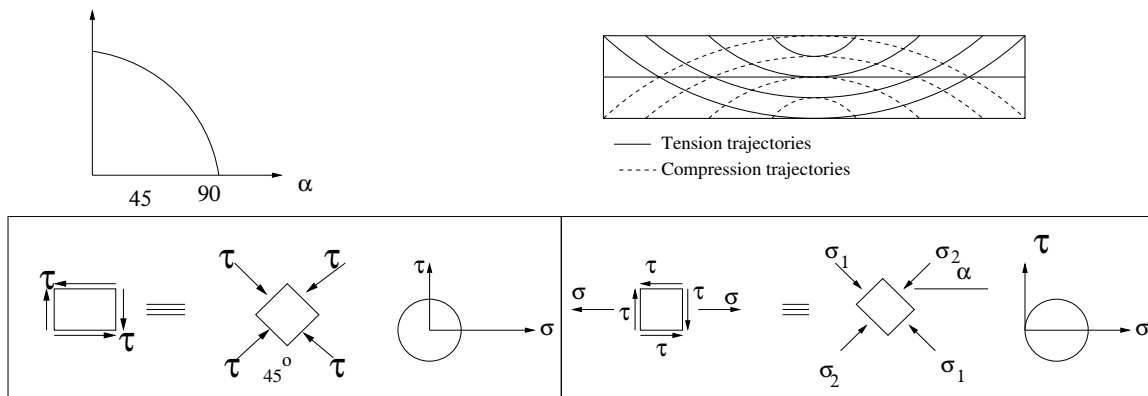


Figure 3.1: Principal Stresses in Beam

- 2 Due to flexure, vertical flexural cracks develop from the bottom fibers.
- 3 As a result of the tensile principal stresses, two types of shear cracks may develop, Fig. 3.2:

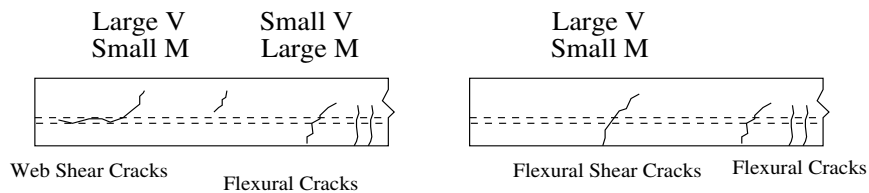


Figure 3.2: Types of Shear Cracks

**Web shear cracks:** Large V, small M. They initiate in the web & spread up & down at  $\approx 45^\circ$ .

3. Compute the principal stresses
  4. Equate principal tensile stress to the tensile strength
- 10 Using a semi-analytical approach
1. Assume that  $f_c$  is directly proportional to steel stress

$$\left. \begin{aligned} f_c &= \alpha \frac{f_s}{n} \\ M_n &= A_s f_s j d \Rightarrow f_s = \frac{M_n}{A_s j d} \end{aligned} \right\} \left. \begin{aligned} f_c &= \alpha \frac{M_n}{n A_s j d} \\ \rho &= \frac{A_s}{b d} \end{aligned} \right\} f_c = \frac{\alpha M_n}{n \rho j b d^2} = F_1 \frac{M_n}{\rho n b d^2} \quad (3.1)$$

2. Shear stress

$$v_n = F_2 \frac{V_n}{b d} \quad (3.2)$$

3. From Mohr's circle, the tensile principal stress is

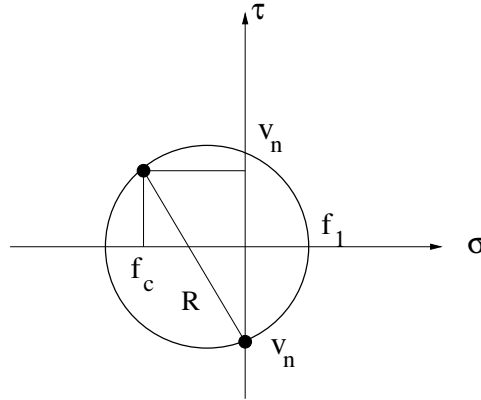


Figure 3.4: Mohr's Circle for Shear Strength of Uncracked Section

$$f_1 = \frac{f_c}{2} + \sqrt{\left(\frac{f_c}{2}\right)^2 + v_n^2} \quad (3.3)$$

4. Set  $f_1$  equal to the tensile strength

$$f_1 = f'_t \Rightarrow f_1 \frac{V_n}{b d} = f'_t \frac{V_n}{b d} \quad (3.4-a)$$

$$\frac{V_n}{b d} = \frac{f'_t V_n}{f_1 b d} \quad (3.4-b)$$

$$= \frac{f'_t}{\frac{f_1 b d}{V_n}} \quad (3.4-c)$$

Combining Eq. 3.1, 3.2, and 3.3

$$\frac{V_n}{b d} = \frac{f'_t}{\left[ \underbrace{\frac{F_1 E_c}{2 E_s} \frac{M_n}{\rho V_n d}}_{C'_1} + \left[ \left( \underbrace{\frac{F_1 E_c}{2 E_s} \frac{M_n}{\rho V_n d}}_{C'_1} \right)^2 + \underbrace{F_2^2}_{C'_2} \right]^{1/2} \right]} \quad (3.5)$$